Abel-Jacobi Condition for Quadrilateral Mesh Generation

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This work discovers the equivalence relation between quadrilateral meshes and meromorphic quartic differentials. Each quad-mesh induces a conformal structure of the surface, and a meromorphic quartic differential, where the configuration of singular vertices corresponds to the divisor of the meromorphic differential. Due to Riemann surface theory, the configuration of singularities of a quad-mesh satisfies the Abel– Jacobi condition. Inversely, if a divisor satisfies the Abel–Jacobi condition, then there exists a meromorphic quartic differential with the given divisor. The meromorphic differential induces a flat metric with cone singularities and special holonomy. Furthermore, if the meromorphic quartic differential is with finite trajectories, then it also induces a quad-mesh, the poles and zeros of the meromorphic differential correspond to the singular vertices of the quad-mesh.